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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL
LABORATORY, HARVARD UNIVERSITY.

EXPERIMENTS ON THE ELECTRICAL OSCILLATIONS OF A HERTZ RECTILINEAR OSCILLATOR.

By George W. Pierce.

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While engaged in calibrating a wavemeter for electric waves, I have made a series of measurements of the wave-length produced by a long Hertz rectilinear oscillator, consisting of two oppositely extending horizontal wires with a spark-gap between. By varying the length of the oscillator, wave-lengths from 16 to 63 meters were obtained. The experiments were conducted in a long room in the third story of the laboratory, so that the oscillator was at a height of 10 meters above the surface of the earth, and represents approximately the conditions that exist when the oscillator is alone in free space.

The experimental results, which give a relation of the wave-length to the length of the oscillator, may be not without interest; because of the existence of numerous very thorough mathematical discussions of the problem.

Apparatus and Plan of the Experiment. — A general idea of the experiment may be had by a reference to Fig. 1, which shows in ground plan the arrangement of the apparatus.

The wavemeter, shown at the left of the figure, consists of a variable condenser C in series with a loop of heavy wire L and a high-frequency electrodynamometer I. The loop of wire L is in the form of a square 30 cm. on a side. The condenser consists of two sets of semicircular plates — one set fixed and the other set movable by rotation about a vertical axis so as to permit variation of capacity by bringing a greater or less area of the two sets of plates into an interlapping position. A scale carried by the top movable plate passes under a fixed pointer, so that the position of the movable plates with respect to the fixed plates can be read after any adjustment of the apparatus.

The high-frequency dynamometer I is of the form previously employed by me in a series of experiments on resonance in wireless telegraph circuits,[1] and consists of a disc of silver, suspended by a quartz fibre, so as to hang near a small coil of a few turns of wire, with the axis of which the plane of the disc makes an angle of 45°, as is shown in Fig. 2. The disc is at M; and the coil, which in this experiment consisted of five turns of wire wound on a vulcanite tube, is shown at C, Fig. 2. The terminals from the coil are connected to binding posts by which the coil is put into the wavemeter circuit. The front of the disc M carries a small mirror, enabling the deflections of the disc to be measured by means of a telescope and scale.

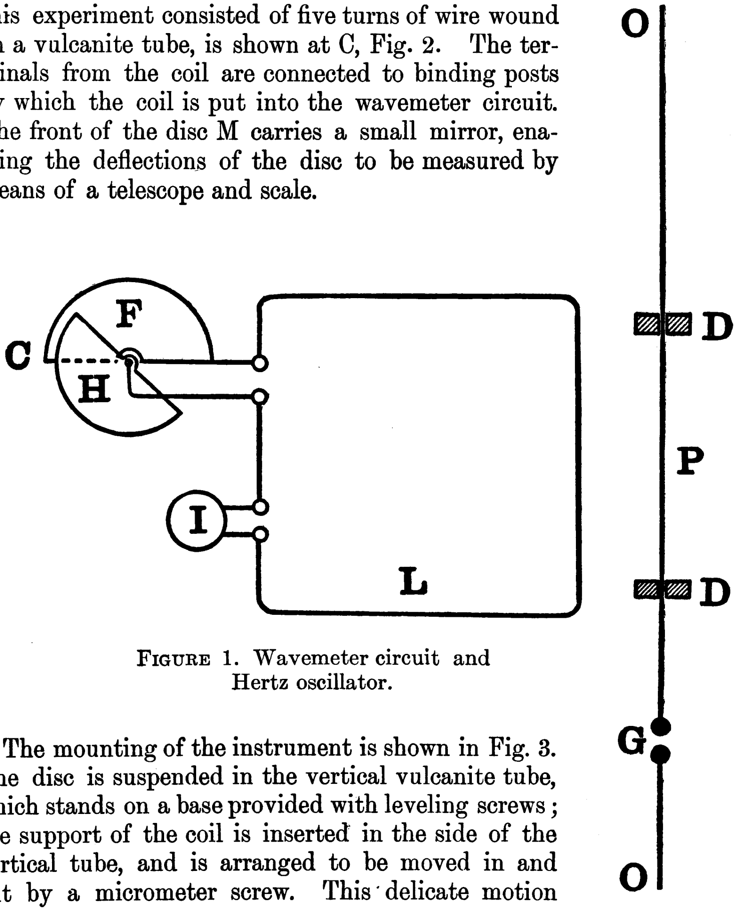

Figure 1. Wavemeter circuit and Hertz oscillator.

The mounting of the instrument is shown in Fig. 3. The disc is suspended in the vertical vulcanite tube, which stands on a base provided with leveling screws; the support of the coil is inserted in the side of the vertical tube, and is arranged to be moved in and out by a micrometer screw. This delicate motion of the coil in or out brings the coil nearer to or farther from the suspended silver disc so as to vary the sensitiveness of the instrument to make it suitable for measuring small or large oscillating currents.

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[1] Phys. Review, 1904, **19**, 196; 1905, **20**, 220; 1905, **21**, 367; 1906, **22**, 159; 1907, **24**, 152.

The action of the instrument is as follows : Oscillations in the coil induce oscillations in the disc, and between these two sets of oscillations there is a force which causes the disc to tend to set itself at right angles to the plane of the coil. A mathematical theory of the instrument, together with some experiments showing that the deflections of the disc are proportional to the square of the current in the coil, is given by me in volume 20, page 226, of the Physical Review for 1905.

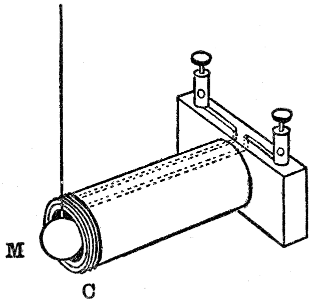

FIGURE 2. Coil and suspended disc of the high-frequency dynamometer.

In place of the dynamometer, a Geissler tube, connected to the two sides of the condenser, was used in some of the experiments.

The Calibration of the Wavemeter. — For wave-lengths greater than 350 meters, I have a set of standard oscillators whose periods have been determined by spark-photographs taken with the revolving mirror.[2] These could, however, not be employed in the present experiments, where the greatest length of oscillator that could be set up in the room had a wave-length of only 63 meters. It was, therefore, necessary to use another method of calibrating the wavemeter of Fig. 1; namely, by tuning it to resonance with an oscillator consisting of various lengths (4 to 17 meters) of two parallel wires, 1 mm. in diameter and 8 cm. apart. It was assumed that the wave-length of such a parallel-wire oscillator is four times the length of one of the wires. This assumption is on the supposition that there is a loop of potential at the free end of the oscillator, and that the velocity of the waves on parallel wires is equal to the velocity of light.

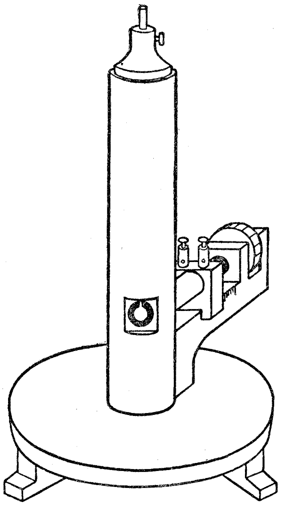

FIGURE 3. Mounting of dynamometer with variable sensitiveness.

In regard to the loop at the free end, Bumpstead[3] has shown that this loop of potential for a parallel-wire oscillator is really beyond the free end by an amount a little less than half the distance apart of the wires. This correction, applied to my experiments, amounts to less than one per cent in the case even of the shortest parallel-wire oscillator used in the calibration, and has been taken into account.

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[2] Phys. Review, 1907, **24**, 152.

[3] Am. Jour. Sci., 1902, **14**, 359.

That the velocity of the waves on the wires is equal to the velocity of light has its theoretical basis in the fact that for rapid oscillations guided by parallel wires, the self-induction per unit of length multiplied by the capacity per unit of length is the reciprocal of the square of the velocity of light. That the velocity of propagation on the parallel wires is the velocity of light has been shown experimentally by Trowbridge and Duane[4] and by Saunders.[5] Recently also Diesselhorst[6] of the Reichsanstalt has made some experiments which indicate that the wave-length on the parallel wires differs from the wave-length in air by less than one-third of one per cent when the parallel wires are not more than 100 meters long.

Wave-length of the Wave Produced by the Hertz Oscillator. — If now we take the two parallel wires, separate them, and extend them out oppositely so as to form a Hertz oscillator, the capacity per unit of length diminishes, while the inductance per unit of length increases. Does the wave-length remain the same; namely, four times the length of the half-oscillator, or $\lambda = 2\,l$, where l is the length of the whole oscillator? Some theoretical writers (Abraham,[7] Rayleigh[8]) say that it does remain very approximately the same (if the diameter of the wire is a small fraction of the length); while, on the other hand, Macdonald[9] has concluded that λ is equal to $2.53\,l$, and he is supported in this conclusion by Pollock and Close.[10]

Experimental tests of the question have heretofore usually been made with very short vibrating systems, to which the theoretical deductions are not directly applicable. A. D. Cole[11] finds $\lambda = 2.52\,l$, for a Klemencic receiver 7 to 8 cm. long and 3.1 mm. in diameter. This is in good agreement with Macdonald's theoretical relation. It is doubtful, however, if Macdonald's equation, which was derived by considering the oscillator or receiver to be indefinitely thin in comparison with its length, was intended to apply to the relatively thick receivers of Cole's experiment.

Another very admirable set of measurements with short oscillators has recently been published by Webb and Woodman.[12] With an untuned receiver they have made measurements of the wave-length produced by rod oscillators of various lengths between 2 and 10 cm., and various diameters between 0.2 and 1.3 cm., and have obtained the wave-length a linear function of the length when the ratio of diameter to length is kept constant, and also the wave-length is a linear function of the ratio of diameter to length when the length is kept constant. By extrapolation from their measured values they find the limiting value of the ratio of the wave-length to the vibrator length, as the diameter approaches zero, to be 2.24.

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[4] Am. Jour. Sci., 1895, **49**, 297.

[5] Phys. Review, 1896, **24**, 152.

[6] Elektrotech. Zeits., 1908, **29**, 703.

[7] Wied. Ann. 1898, **66**, 435.

[8] Phil. Mag., 1904, **8**, 105.

[9] Electric Waves, 111.

[10] Phil. Mag. 1904, **7**, 635.

[11] Phys. Review, 1905, **20**, 268.

[12] Phys. Review, 1909, **29**, 89.

Coming now to the experiments that have been made with the longer oscillators, I find two measurements mentioned by Drude[13] in which he obtains for a wire 1 mm. in diameter and 4 meters long the wave-length 8.42 meters, and for a wire 2.5 meters long the wave-length 5.24. *These two experiments give* $\lambda = 2.10\ l$.

Also there is a series of measurements by F. Conrat[14] for rectilinear oscillators 2 to 6 meters long (1 mm. diameter). These measurements are presented in Table I., and *show the average relation* $\lambda = 2.12\ l$.

TABLE I.

Conrat's Values for Relation of λ to l.

l. Length of Oscillator in Meters.	λ. Wave-length in Meters.	λ/l.
2.00	4.20	2.10
3.84	8.00	2.09
4.00	8.40	2.10
5.50	12.00	2.18
6.30	13.40	2.12
Average		2.12

My measurements, extending the experimental records in the direction of the longer waves, are given in Table II. The diameter of the wire employed was 1 mm. The result obtained is that *the wave-length of the oscillator is* 2.094 *times its length.* This is in good agreement with the results obtained by Drude and in fair agreement with those of Conrat.

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[13] Ann. d. Physik, 1903, **11**, 965.

[14] Ibid., 1907, **22**, 670.

Taking the present observation together with those of Drude and of Conrat it appears that the wave-length of a Hertz rectilinear is very close to 2.10 times the length of the oscillator, provided the oscillator is not less than two meters long and is of comparatively small diameter. The influence of the diameter in determining the wave-length was not tested further than by a single observation, in which it was found that an oscillator made of two brass tubes, each 6 meters long and 22 mm. in diameter, had a wave-length 2.14 times its length.

TABLE II.

RESULTS OBTAINED IN PRESENT EXPERIMENT.

l. Length of Oscillator in Meters.	Wave-length in Meters.	λ/l.
8.0	16.9	2.11
9.0	18.9	2.10
10.0	21.2	2.11
11.0	23.2	2.11
12.0	24.9	2.08
14.0	29.5	2.10
16.0	33.6	2.10
18.0	38.1	2.11
20.0	41.6	2.08
22.0	46.1	2.11
24.0	49.5	2.07
26.0	53.9	2.07
28.0	57.5	2.06
30.0	63.0	2.10
Average		2.094

Comparison of the Result with Abraham's Theoretical Relation. — The value obtained theoretically by Abraham, as a second approximation for the wave-length of a thin rod in terms of its length and diameter, is

$$\lambda = 2\,l(1 + 5.6\,\epsilon^2),$$

where

$$\epsilon = \frac{1}{4 \log_e \frac{2\,l}{d}},$$

in which l is the length of the whole oscillator, and d its diameter. The formula was derived by applying Maxwell's equations to a long, perfectly conductive ellipsoid of revolution, and taking the limit approached by λ when the square of the minor axis of the ellipsoid vanishes in comparison with the square of the major axis. Under these conditions the major axis becomes the length of the rod-oscillator and the minor axis its diameter.

To show the size of the $5.6\,\epsilon^2$ term of Abraham's formula, the following table (Table III.) has been computed for various values of l/d, covering the range of the experiments by Webb and Woodman and those by Conrat and by me.

TABLE III.

Computation of the $5.6\epsilon^2$ Term of Abraham's Formula.

l/d.	$5.6\epsilon^2$	Range.
4	.081	Webb and Woodman.
5	.065	
7	.050	
10	.039	
160	.011	
2000	.005	Conrat.
4000	.0043	
6000	.0039	
8000	.0037	Writer.
12000	.0035	
20000	.0031	
30000	.0029	

It is seen that in the range of my experiments, the $5.6\,\epsilon^2$ term raises the theoretical value of the wave-length to $2.006\,l$, and in Conrat's range to $2.01\,l$. This term is, therefore, entirely inadequate to account for the 5 per cent excess of the experimental values over the theoretical values of Abraham.

Also the presence of the spark-gap in the oscillator seems to be without influence, as the values of Conrat were obtained for rods without a gap.

In discussing the question, raised by Pollock and Close,[15] as to whether a result obtained for an infinitely thin ellipsoid can be applied to an infinitely thin rod of uniform section, Lord Rayleigh[16] says: "It appears therefore that the wave-length of the electrical vibration associated with a straight terminated rod of infinitesimal section is equal to twice the length of the rod, whether the shape be cylindrical so that the radius is constant, or ellipsoidal so that the radius varies in a finite ratio at different points of the length, and that this conclusion remains undisturbed, even though the shape be not one of revolution." Lord Rayleigh, however, raises the question whether a sufficient reduction of the diameter of the rod to comply with Abraham's approximation is experimentally possible without too greatly diminishing the conductivity, which is assumed perfect in the theoretical discussion.

In reply to this note by Lord Rayleigh, Macdonald[17] expresses the view that the rate of damping of the free vibration associated with the terminated straight wire is very large, and in fact not far removed from the order of magnitude of the known result for a spherical vibrator. This large damping, if it exists, and especially if it is due to a large radiation from the wire near the ends, would account for a distortion of the current distribution in the conductor so as to give a wave-length larger than twice the length of the conductor.

Since the question of the conductivity of the wire and the damping of the oscillations has a bearing on the question of its period, it is proposed to give the results of a measurement made on the damping of one of the oscillators used in the present experiments.

Damping. — The damping factor of a rectilinear oscillator 14 meters long, consisting of two oppositely-extending horizontal wires 7 meters long and 1 mm. in diameter, was determined by a method recently given by K. E. F. Schmidt.[18] The spark-gap was 3 mm. long. Schmidt's method consists in determining the average square current in a low resistance wavemeter circuit for various adjustments of the wavemeter in the neighborhood of resonance. To get the mean square current in the wavemeter circuit the dynamometer shown in Figs. 2 and 3 was employed. The deflections of this instrument have been shown to be proportional to the square of the current. The values obtained are recorded in Table IV, which gives the wave-length adjustment of the wavemeter and the corresponding relative deflections of the dynamometer.

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[15] Loc. cit.

[16] Loc. cit.

[17] Phil. Mag., 1904, **8**, 276.

[18] Phys. Zeits., 1908, **9**, 13.

TABLE IV.

For Determining Damping.

Adjustment of Wavemeter. λ in Meters.	λ/λ_m.	D/D_m. Deflection relative to Maximum.
30.3	.992	.99
29.2	.960	.58
27.6	.908	.160
26.0	.859	.054
32.2	1.062	.39
33.9	1.111	.150
31.5	1.032	.71
30.5	1.000	1.00

These results are plotted in the curve of Fig. 4, in which the abscissas are λ/λ_m, and the ordinates D/D_m.

Schmidt's method of getting the damping from this curve consists in determining the width between the two branches of the curve at ordinates .55, .70, and .85, and then making use of a decrement diagram which he has computed and plotted in his original paper, to which the reader is referred. This method applied to the present case gives the values in Table V.

TABLE V.

Decrement by Schmidt's Method.

Ordinate.	Width of Res. Curve, reduced to Proper Scale.	δ
.55	.88	.32
.70	.64	.33
.85	.40	.32

The last column of this table gives the logarithmic decrement per complete oscillation. The value .32, including the Joulean decrement as well as the radiation decrement, is 40 per cent higher than the logarithmic decrement due to radiation alone, as computed by Abraham's formula for the decrement, which is

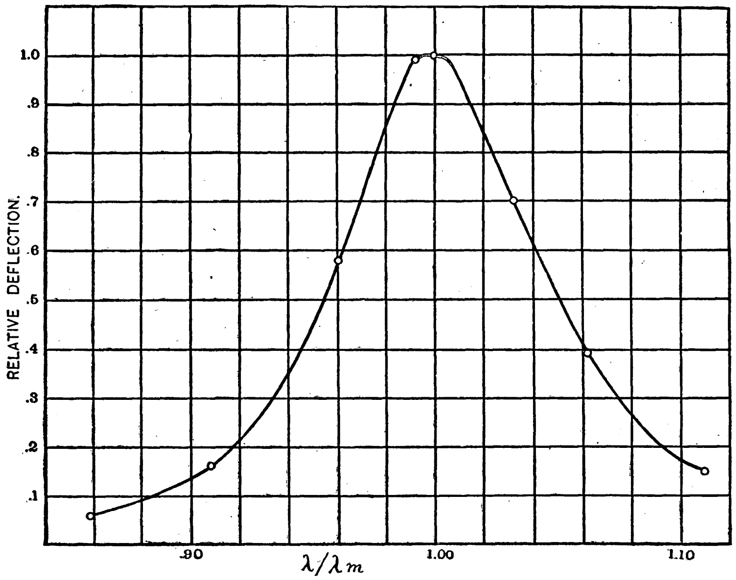

FIGURE 4. Resonance curve used in obtaining logarithmic decrement.

$$\delta = \frac{9.74}{4\log_e \frac{2l}{d}}$$
$$= .23$$

The value of the decrement is, however, too small to produce a change in the measured value of the wave-length by more than a small fraction of one per cent.

Summary of Results. — Assuming that the wave-length produced by the parallel-wire oscillator is four times the length of one of the wires, the wave-length produced by the fundamental electrical vibration of a long, thin, rectilinear Hertz oscillator was found to be 2.094 times the total length of the oscillator, for oscillators of length between 8 and 30 meters.

This result is 4.5 per cent higher than Abraham's theoretical value computed by the formulas

$$\lambda = 2\,l(1 + 5.6\,\epsilon^2)$$

$$\epsilon = \frac{1}{4 \log_e \frac{2\,l}{d}}.$$

The results obtained in the present experiments are in approximate agreement with two measurements given by Drude and with a series of measurements obtained by Conrat, both using oscillators of length between 2 and 6 meters.

Jefferson Physical Laboratory,
Cambridge, Mass.,
December, 1909.